Worksheet on Limits and ϵ - δ Proofs:
GSI: Chris Peterson

Problem 1: Consider the function $f(x)$ defined by the following two cases:

$f(x) = \frac{1}{n+1}$ for $x \in (\frac{1}{n+1}, \frac{1}{n}]$ where $n = 1, 2, 3, \ldots$

$f(x) = 1$ for all other x.

Draw a graph of this function and determine $lim_{x \to 0^+} f(x)$ (the limit as f approaches 0 from the right). Use both an ϵ - δ argument and a Squeeze Theorem argument to show that your answer is correct.

Problem 2: Determine the following limits (if they exist):

- $lim_{x \to 6}(\frac{\sqrt{x+3}-3}{x-6})$
- $lim_{x \to 3}(\frac{\sqrt{x-2}}{x-3})$
- $lim_{x \to -12}(\frac{\sqrt{x^2+25}-13}{x+12})$ (Similar to 2.3.30)

Problem 3: For any positive integer n, the formula $f(x) = nx$ determines a line with slope n. Use an ϵ - δ argument to show that $lim_{x \to 1}(nx) = n$. How does the relationship between ϵ and δ change as n increases? Use a graph to illustrate this dynamic.

Problem 4: Determine $lim_{x \to 4} \frac{x-3}{(x-4)^2}$. Prove that your answer is correct. (You may use either an ϵ - δ type proof or a Squeeze Theorem argument.)

Problem 5 (Challenge): We know that limits of the function $f(x) = x^2$ can be quickly determined using the Direct Substitution Property. Use an ϵ - δ argument to show that $lim_{x \to c}(f(x)) = c^2$ for any real number c. How does the relationship between ϵ and δ change as c moves away from zero?